where $C \gg 1$. For certain fuels we may take $C = \infty$, and then Eq. (2) simplifies; the terms containing y_2 vanish.

By way of example, Table 1 gives the approximation coefficients for $O_2 - H_2$ fuel and four values of the excess coefficient of the oxidizing agent. The quantities in this table were determined by the well-known method employed for the analytical representation of empirical data [6]. In the present example $C = \infty$.

The results of our determination of ξ_n for oxygen—hydrogen fuel with $\alpha = 1$ and $r_a/r_{cr} = 10$ by means of Eq. (2) and also by numerical integration of the problem are presented in Fig. 1. These data, together with the results of analogous calculations carried out for $p_k = 0.5$-25 MN/m², $r_{cr} = (2.5$-$125)\ 10^{-3}$ m and $r_a/r_{cr} = 3$-15 in the case of fuels containing hydrogen, carbon, nitrogen, oxygen, and fluorine, show that the maximum error in the determination of losses due to the lack of chemical equilibrium in the flow through the nozzle by means of the proposed approximate equation equals ±0.002. Such an error in ξ_n introduces an error not exceeding 0.2% into the determination of the specific momentum.

NOTATION

ξ_n, loss coefficient of the specific momentum due to the lack of chemical equilibrium in the flow; r_{cr}, radius of critical nozzle cross section; r_a, radius of nozzle outlet section; p_k, flow retardation pressure; p_0, normal pressure; h, relaxation coefficient; ξ_0, m, n, s, C, dimensionless coefficients; α, excess coefficient of oxidizing agent.

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COUPLED HEAT TRANSFER BETWEEN FLUID FILM AND SOLID WALL

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A numerical algorithm is proposed for solution of the coupled problem of convective heat transfer. The method was used to study two-phase heat transfer between a solid wall and a laminarly flowing fluid film for a linear temperature profile at the outer surface of the wall. A computational formula is proposed for the dimensionless Nusselt number.

In studying heat transfer in a film flowing gravitationally along a wall, the temperature at the solid–fluid-film interface is usually assumed known and equal to a given temperature at the outer surface of the wall. This condition is satisfied in the extremely idealized case of a wall with infinitely large thermal conductivity. However, the coefficients of thermal conductivity for several polymer materials such as Teflon and vinyl are of the

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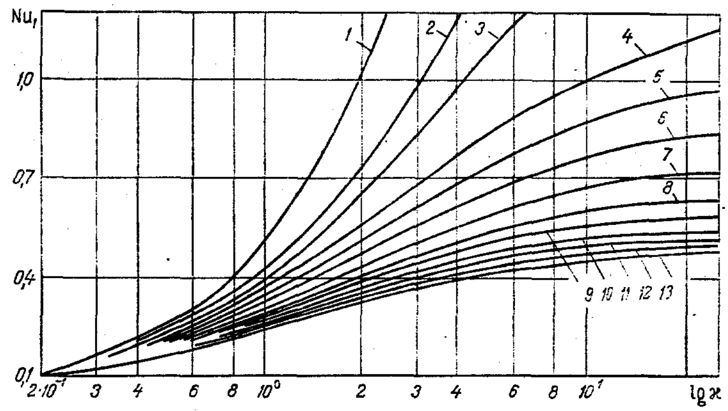

Fig. 1. Dependence of dimensionless Nusselt number on $\varkappa$ for various values of the parameter P: 1) log P = $-\infty$; 2) log P = -2; 3) log P = -1.5; 4) log P = -1; 5) log P = -0.8; 6) log P = -0.6; 7) log P = -0.4; 8) log P = -0.2; 9) log P = 0; 10) log P = 0.2; 11) log P = 0.4; 12) log P = 0.6; 13) log P = $+\infty$.

same order of magnitude as that of the fluid. In the general case, such problems must be considered as coupled, where the energy equations in the fluid and solid are solved simultaneously using the velocity distribution in the flowing film with the temperatures and thermal fluxes being assumed equal at the interphase boundary, i.e., using boundary conditions of the fourth kind [1-3].

Such formulations of the problems were discussed in a number of papers both as internal and external problems [5, 6]. Analytic methods were also proposed recently for solution of the coupled problems [7, 8].

In this paper, a numerical algorithm is proposed for solution of the coupled problem of convective heat transfer. A fluid film flows gravitationally along the surface of a vertical, infinitely wide plate of thickness b and length l. We denote the thickness of the flowing laminar fluid film by h. We choose a Cartesian coordinate system such that the x axis coincides with the interface with x = 0 at the upper end of the plate and x = l at the lower end. At the outer side of the plate (y = $-$b) and also on the free surface of the film (y = h), known temperature profiles Ψ_2 and Ψ_1 are assigned, respectively. In the general case, the energy equations and boundary conditions in the selected coordinate system have the form

$$3U\left(\frac{y}{h}-\frac{1}{2}\left(\frac{y}{h}\right)^2\right)\frac{\partial t_1}{\partial x}=a_1\left(\frac{\partial^2 t_1}{\partial y^2}\right);\qquad \frac{\partial^2 t_2}{\partial x^2}+\frac{\partial^2 t_2}{\partial y^2}=0; \tag{1}$$

$$t_1=\Psi_1(x)\ \text{for}\ y=h;\ t_2=\Psi_2(x)\ \text{for}\ y=-b;\qquad t_1=\varphi_1(y),\ t_2=\varphi_2(y)\ \text{for}\ x=0,\ t_2=\Theta_2\ \text{for}\ x=l; \tag{2}$$

$$t_1(x)=t_2(x),\ \lambda_1\frac{\partial t_1}{\partial y}=\lambda_2\frac{\partial t_2}{\partial y}\ \text{for}\ y=0. \tag{3}$$

We investigate the solution of the problem (1)-(3) under simplified boundary conditions where $\Psi_1 = \varphi_1 = \varphi_2 = t_0$ = const, $\Theta_2 = t_1$, and the function $\Psi_2(x)$ varies linearly from t_0 at x = 0 to t_1 at x = l, i.e., $\Psi_2(x) = t_0 + (t_1 - t_0)(x/l)$.

These simplifications, without changing the generality of the numerical realization of the solution for the problem of coupled convective heat transfer, can correspond to heating (cooling) of the fluid film or to vapor condensation on the solid surface with subsequent flow in film form if the effect of a change in the thickness of the latter on heat transfer can be neglected.

We convert to the dimensionless variables

$$T_1=\frac{t_1-t_0}{t_l-t_0},\ \eta_1=1-\frac{y}{h},$$

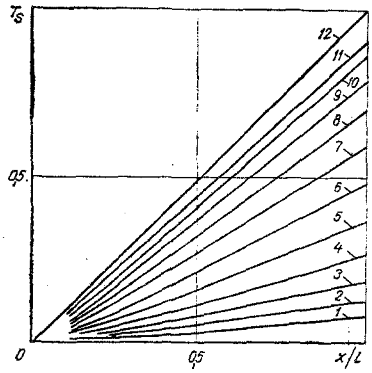

Fig. 2. Dependence of dimensionless temperature at the phase interface on the dimensionless length x/l for log P = 0 and various values of the parameter $\varkappa$: 1) $\varkappa$ = 0.1; 2) $\varkappa$ = 0.16; 3) $\varkappa$ = 0.25; 4) $\varkappa$ = 0.4; 5) $\varkappa$ = 0.63; 6) $\varkappa$ = 1.0; 7) $\varkappa$ = 1.6; 8) $\varkappa$ = 2.5; 9) $\varkappa$ = 4.0; 10) $\varkappa$ = 6.3; 11) $\varkappa$ = 10.0; 12) $\varkappa$ = ∞.

in the liquid phase and define the corresponding variables in the solid phase in the form

$$T_2 = \frac{t_2 - t_0}{t_l - t_0}, \quad \eta_2 = 1 + \frac{y}{b}.$$

In addition, we introduce the dimensionless length $z = x/l$ along the x axis. After introduction of the dimensionless variables, we rewrite the problem (1)-(3) in the following manner:

$$3\left(\eta_1 - \frac{1}{2}\eta_1^2\right)\frac{\partial T_1}{\partial z} = P\frac{\partial^2 T_1}{\partial \eta_1^2};$$
$$\frac{\partial^2 T_2}{\partial z^2} + S\frac{\partial^2 T_2}{\partial \eta_2^2} = 0; \tag{4}$$

$$T_1 = 0 \text{ for } \eta_1 = 0; \; T_2 = z \text{ for } \eta_2 = 0;$$
$$T_1 = T_2 = 0 \text{ for } z = 0; \; T_2 = 1 \text{ for } z = 1; \tag{5}$$

$$T_1 = T_2, \; \frac{\partial T_1}{\partial \eta_1} = -\varkappa\frac{\partial T_2}{\partial \eta_2} \text{ for } \eta_1 = \eta_2 = 1, \tag{6}$$

where P, S, and $\varkappa$ are dimensionless parameters. For numerical solution of this problem by the mesh method, we selected a set of points with the coordinates $\eta_i = (1/2 + m)d$, $z = (1/2 + n)d$, where n, m = −1, ..., M and d = 1/M, i = 1, 2. The derivatives in Eqs. (4) were replaced by central differences using a four-point implicit scheme. The resultant difference analog was solved by the sweep method for parabolic equations and by the vector sweep method for elliptic equations. To better approximate the boundary conditions in the fluid and gas phases, two fictitious points, (−h/2, 1 + h/2), falling outside the segment [0, 1], were introduced for the variable η. The temperature at the interface between the fluid film and the solid wall was calculated as the mean value of the temperatures at the points M − 1 and M:

$$f(z) = (T_{1,M} + T_{1,M-1})/2 = (T_{2,M} + T_{2,M-1})/2, \tag{7}$$

where the second subscript in the temperature notation corresponds to the labelling of the point. One of the basic difficulties in the solution of the coupled problems lies in the resolution of boundary conditions of the fourth kind, i.e., the conditions for coupling parabolic equations with elliptic equations. There are a number of practical methods for solving such problems [3, 7, 8]. In this report, we propose to accomplish this in the following way.

We express the temperature f(z) at the interface through the temperatures at the points M − 2, M − 1, and M:

$$f(z) = \frac{-T_{1,M-2} - 9T_{1,M} + 18T_{1,M-1} - \varkappa T_{2,M-2} + 18T_{2,M-1} - 9\varkappa T_{2,M}}{8 + 8\varkappa}. \tag{8}$$

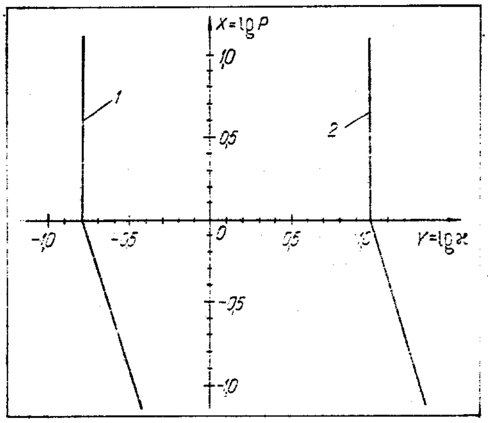

Fig. 3. X − Y plane, X = log P, Y = log $\varkappa$. At points to the left of curve 1, the resistance to heat transfer is concentrated in the solid phase. At points to the right of curve 2, the resistance is concentrated in the liquid phase.

In writing down Eq. (8), the coupling conditions (6) were used where the derivatives were replaced by difference analogs, which, for example, have the form

$$\partial T_1/\partial \eta_1 = (T_{1,M-1} + 9T_{1,M} - 18T_{1,M-1} + 8f)/(12h) + 0(h^3)$$

for the liquid phase. Since this problem is solved by an iteration method, the temperature at the phase interface is assigned arbitrarily in the first iteration. Let the temperature at the interface be $f_i(z)$ after the i-th iteration. Adjustment factors, and consequently the temperature fields, are determined by means of Eq. (7) and the boundary conditions (5) and (6). A new temperature at the interface, i.e., the function $f_{i+1}(z)$, is determined through Eq. (8) from the temperature fields found for the fluid film and solid, respectively. The calculation is considered finished if the inequality $|f_{i+1}(z) - f_i(z)| < 0.05$ is satisfield.

The proposed numerical algorithm was used to solve the problem (4)-(6) on a BÉSM-6 computer. Computing time for a single variant and a step d = 0.1 was of the order of one minute.

We define the dimensionless Nusselt numbers for the first and second phases in the following manner:

$$I = (\lambda_1 (t_l - t_0) l/h) \int_0^1 \frac{\partial T_1}{\partial \eta_1} dz = (\lambda_2 (t_l - t_0) l/b) \int_0^1 \frac{\partial T_2}{\partial \eta_2} dz. \tag{9}$$

Between $\mathrm{Nu}_1 = \int_0^1 \frac{\partial T_1}{\partial \eta_1} dz$ and $\mathrm{Nu}_2 = \int_0^1 \frac{\partial T_2}{\partial \eta_2} dz$ there is the relation

$$\mathrm{Nu}_1 = \varkappa \mathrm{Nu}_2. \tag{10}$$

In the general case, the solution of the problem (4)-(6) depends on the three independent parameters S, P, and $\varkappa$. However, the effect of S can be neglected if the inequality $S \gg 1$ is satisfied [for actual tubing of length l meters, $b = \delta \cdot 10^{-3}$ m and $S = (l \cdot 10^3/\delta)^2 \gg 1$ in order of magnitude, where l and δ are of the order of one]. Consequently, the solution of the problem (4)-(6) depends on the two parameters p and $\varkappa$ under actual geometric dimensions.

Calculated results for the dimensionless number Nu_1 as a function of $\varkappa$ are shown in Fig. 1 for various values of the parameter P. For any fixed value of the parameter P there exist numbers $\varkappa_{min}$ and $\varkappa_{max}$ such that the relation

$$\mathrm{Nu}_1(\varkappa) = \varkappa/2 \tag{11}$$

is valid when the inequality $\varkappa \le \varkappa_{min}$ is satisfied. Graphically, this means that the curves $\mathrm{Nu}_1(\varkappa)$, for sufficiently small $\varkappa$, practically coincide with the limiting curve 1 in Fig. 1 for which log P = −∞ (we limit ourselves to 10% accuracy in the following). Then the surface temperature $T_s(z) = 0$ at any point on the interface. The physical significance of Eq. (11) is clear from the definitions (9) and (10); when $T_s = 0$, the equality $\partial T_2/\partial \eta_2 = z$ is valid and therefore

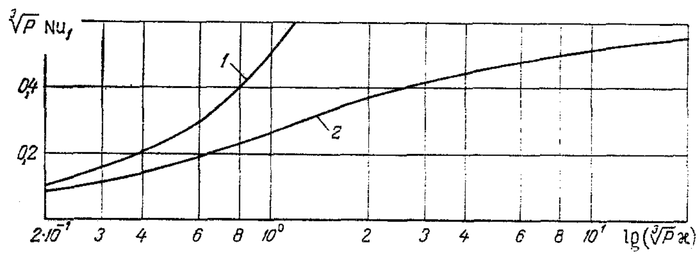

Fig. 4. Dependence of dimensionless function $P^{1/3}Nu_1$ on the variable $P^{1/3}\varkappa$ for log $P \leq 0$ (curve 2). Curve 1 is the function $P^{1/3}\varkappa/2$.

$$\mathrm{Nu}_2(\varkappa) = \frac{1}{2} \quad \text{or} \quad \mathrm{Nu}_1 = \frac{\varkappa}{2}. \tag{12}$$

Equation (12) means that the resistance to heat transfer is totally concentrated in the solid phase.

At large values of $\varkappa$, the function $Nu_1(\varkappa)$ tends to a limiting value and differs from it by less than 10% when $\varkappa \geq \varkappa_{max}$. In this case, the temperature at the phase interface agrees with the assigned temperature at the outside of the wall. Resistance to heat transfer is concentrated in the fluid and the problem can only be solved in the film under the boundary condition $T_S(z) = \Psi_2$. The case $\Psi_2 = z$ is discussed here. It is obvious that the properties indicated above will also be valid for an arbitrary function $\Psi_2(z)$. Calculations of the function $T_S(z)$ for a number of values of $\varkappa$ in the interval $(\varkappa_{min}, \varkappa_{max})$ are shown in Fig. 2 for log $P = 0$.

Values of $\varkappa_{min}$ and $\varkappa_{max}$ can be obtained from Fig. 3 which is a plot of the (X — Y) plane where $X = \log\varkappa$ and $Y = \log P$ with X_{min} and X_{max} being approximated in the following manner:

$$X_{\min} = -0.8, \quad X_{\max} = 1 \quad \text{for} \quad \lg P \geqslant 0,$$
$$X_{\min} = -0.8 - \frac{\lg P}{3}, \; X_{\max} = 1 - \frac{\lg P}{3} \quad \text{for} \quad \lg P < 0. \tag{13}$$

As is clear from Fig. 1, in the region of the X — Y plane where log $P \geq 0.6$, the functions $Nu_1(\varkappa)$ corresponding to different P cease to depend on the parameter P and practically concide with the limiting curve 13 for which log $P = \infty$. When log $P = \infty$, the film temperature for any value of z is

$$T_1(z, \eta_1) = \eta_1 T_S(z), \quad \text{where} \quad T_S(z) = \frac{\varkappa}{1+\varkappa} z. \tag{14}$$

Substituting the distribution (14) into Eq. (9), the analytic relation

$$\mathrm{Nu}_1(\varkappa) = \frac{1}{2}\left(\frac{\varkappa}{1+\varkappa}\right) \tag{15}$$

can be obtained for the Nusselt number for the limiting curve 13. For small values of P, there is a thin thermal boundary layer $\sqrt[3]{Pz} \ll 1$ in the neighborhood of the solid wall. In this case, one can show that the equations

$$T_S(z) = (\alpha\varkappa z \sqrt[3]{Pz})/(1 + \alpha\varkappa \sqrt[3]{Pz}),$$
$$\mathrm{Nu}_1(\varkappa) = F(\varkappa \sqrt[3]{P})/\sqrt[3]{P}, \tag{16}$$

where $\alpha = 1$ as shown by calculation, are valid. Consequently, if one plots the quantity $P^{1/3}Nu_1$ on the ordinate and the quantity $\varkappa P^{1/3}$ on the abscissa, one merely need know the function $F(\varkappa P^{1/3})$ in order to obtain a solution. It is shown in Fig. 4 (curve 2). The equality $P^{1/3}Nu_1 \simeq P^{1/3}\varkappa/2$, which is equivalent to Eq. (11), is satisfied when $\log(\varkappa P^{1/3}) \leq -0.8$. The function $P^{1/3}Nu_1 \to 0.6$ when $\log(\varkappa P^{1/3}) \geq 1$. As shown by calculated results, Eq. (16) is valid everywhere when log $P \leq 0$. Consequently, when log $P \leq 0$, the limiting points X_{min} and X_{max} in the X — Y plane must be located on the lines $X = -0.8 - Y/3$ and $X = 1 - Y/3$ as is evident from Eq. (13) and Fig. 3.

Since the solution of the problem (4)-(6) for log $P \leq 0$ is self-similar as shown in Fig. 4 and the maximum difference between curves in Fig. 1 for log $P \geq 0$ is 20%, the dimensionless Nusselt function is approximated within 10% for any $\varkappa$ and P by the following equations:

$$\mathrm{Nu}_1 = \begin{cases} 0.55\left(\frac{\varkappa}{1+\varkappa}\right) & \text{for } \lg P \geqslant 0, \\ 0.55\left(\frac{\varkappa\sqrt[3]{P}}{1+\varkappa\sqrt[3]{P}}\right)\left(\frac{1}{\sqrt[3]{P}}\right) & \text{for } \lg P \leqslant 0. \end{cases} \tag{17}$$

We make some estimates of the parameters P and $\varkappa$ in order to see what region of the X—Y plane (Fig. 3) corresponds to actual values of the Reynolds number.

Film motion can be considered laminar if Re < 400 and then film thickness is determined by the Nusselt formula $h = 1.45\Theta(\mathrm{Re})^{1/3}$, where $\Theta = (\nu_1^2/g)^{1/3}$ [9]. The thermophysical characteristics of water and organic fluids are constant over a broad range of temperatures (180°C > t > 20°C) and are, in order of magnitude, $\lambda_1 \simeq 0.4$N/(sec·deg), $C_p \simeq 4000$ J/(kg·deg), and $a_1 \simeq 10^{-7}$ m²/sec. Only viscosity undergoes a marked variation with temperature with Θ varying from $7\cdot10^{-5}$ m at 20°C to 10^{-5} m at 180°C. For irrigation channels with dimensions $l > 0.1$ m, $b = \delta\cdot10^{-3}$ m ($\delta > 1$), the order of magnitude of P is $P \sim (10^2 l/\Theta)/(\mathrm{Re})^{4/3}$, P(0.05-$10^5$). The value of the second parameter $\varkappa$ also varies over a broad range and can take on any value in the interval ($\varkappa_{min}$, $\varkappa_{max}$) and outside it. For example, where $\varkappa \sim 6(\Theta\cdot10^5)(\mathrm{Re})^{1/3}/\delta$ for metal tubing with $\lambda_2 \sim 40$ N/(sec·deg), $\varkappa \sim 3(\Theta\cdot10^3)(\mathrm{Re}^{1/3})/\delta$ for such materials as Teflon and vinyl with $\lambda_2 \sim 0.2$ N/(sec·deg). Thus, it is often necessary to consider the thermal interaction of the phases even for laminar motion in the film flow (Re < 400).

NOTATION

t_1, t_2, temperatures; T_1, T_2, dimensionless temperatures; λ_1, λ_2, coefficients of thermal conductivity; x, y, spatial coordinates; l, plate length; b, plate thickness; h, film thickness; η_1, η_2, $z = x/l$, dimensionless coordinates; a_1, coefficient of thermal diffusivity; ν_1, kinematic viscosity; g, gravitational acceleration; U, mean fluid velocity; dimensionless parameters: $P = l/(h\mathrm{Pe})$, $S = (l/b)^2$, $\varkappa = (\lambda_2 h)/(\lambda_1 b)$; Nu_1, Nu_2, Nusselt numbers; q, irrigation density; $\mathrm{Re} = q/\nu_1$, Reynolds number; $\mathrm{Pe} = q/a_1$, Peclet number; $I = \lambda_1 \int_0^l (\partial t_1/\partial y)dx$, heat flux across phase interface. Indices: 1, liquid phase; 2, solid phase.

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